case study#4 – Joint Analysis of Rayleigh & Love waves (+ HVSR)

**Goal:** to provide evidence of the fact that fundamental mode of Rayleigh waves can be practically completely absent thus showing the importance of joint analysis with Love waves.

The presented *dataset* was acquired for geotechnical purposes over a slope of a hill in Tuscany (Italy). Data appear extremely interesting cause they put in evidence the fact that, under some conditions, Love waves are essential to clarify and properly interpret Rayleigh-wave dispersion. Acquired data are reported in Figure 1. It can be easily noted that in the 20-40Hz frequency range (pertaining to the shallowest layers), phase velocities of the signal dominating the Rayleigh-wave velocity spectrum are clearly higher (about 400 m/s) with respect to the ones shown by the Love-wave velocity spectrum (about 250 m/s). Although the dispersion curves of Rayleigh and Love waves are clearly different, such a large difference should alarm, even if a "newcomer" would surely consider the signal dominating Rayleigh waves as related to the fundamental mode.

With the aim of illustrating the use of Pareto front analysis in a joint inversion procedure, we analyzed the data according to two different interpretative hypotheses.

**Hypothesis#1** (the one that would have been followed by non-experts): the signal dominating the Rayleigh-wave velocity spectrum is interpreted as belonging to the fundamental mode.

**Hypothesis#2:** the dominating signal is attributed to the first higher mode while the small signal in the 25-30Hz range with a phase velocity of about 250 m/s to the fundamental mode (please notice that we could also avoid this second "attribution" thus considering the first higher mode only).

Results of the (automatic) inversion performed while considering the first hypothesis are reported in Figure 2 and 3. Please notice two facts:

1. the dispersion curves of the Pareto models (i.e. the best models) do not match the picked ones (Figure 2). Rayleigh-wave dispersion curves tend to stay below (i.e. be slower than) the picked one (for Love waves the opposite occurs).
2. the model distribution in the bi-objective space (Figure 3) is clearly "unbalanced": the cloud of models does not point towards the [0, 0] utopia point and the Pareto-front models are not symmetric when compared to the universe of considered models (for details please see the papers reported in the References).

![Figure 1. Upper panel: Rayleigh waves (RVF component); lower panel: Love waves (THF).](image-url)
Figure 2. First interpretative hypothesis: the signal dominating Rayleigh waves in the 20-55Hz frequency range (phase velocity approximately equal to 400 m/s) is attributed to the fundamental mode.

Figure 3. First hypothesis: model distribution in the bi-objective space (obj#1 represent Rayleigh-wave misfit; obj#2 relates to Love waves). The cloud of models does not point towards the [0, 0] utopia point and the Pareto optimal models (red circles) are not symmetric with respect to the universe of evaluated models (symmetry index equal to 0.13 – see Dal Moro and Ferigo, 2011).
Figures 4 and 5 report the main outcomes of the joint inversion performed while considering the second interpretative hypothesis (the energy dominating the Rayleigh-wave velocity spectrum is now interpreted as belonging to the first higher mode).

Now the dispersion curves of the model belonging to the Pareto front perfectly overlap with the picked dispersion curves (Figure 4) for both Rayleigh and Love waves. Moreover, the model distribution in the bi-objective space coherently points towards the utopia point (Figure 5). Please also notice that now the $V_S$ profiles of the Pareto front models (right panel in Figure 4) appear extremely so-to-say focused (compare with Figure 2) showing two main horizons: the first at a depth of about 5m and the second 10m.

**Figure 4.** Second interpretative hypothesis: the energy dominating the Rayleigh-wave spectrum in the 20-55Hz frequency range is interpreted as belonging to the first higher mode.

**Figure 5.** Second hypothesis: model distribution in the bi-objective space. Now the cloud of models points towards the utopia point and the Pareto optimal models (red circles) are highly symmetric with respect to the universe of evaluated models (symmetry index equal to 0.95).
The goodness of the second hypothesis is supported by two further facts.

**Fact#1 - CPT data**

Figure 6 reports the CPT (Cone Penetration Test) that stopped at about 6m because of a stiff layer that, in the light of the surface wave analysis, can be interpreted as a gravel-like material while the real bedrock is actually deeper (10m depth - compare with the V_s profile in Figure 4).

![Figure 6](image)

**Figure 6. Cone Penetration Test**: at a depth of about 6 meters, CPT stopped because of a stiff layer. Compare with the horizon put in evidence by the joint analysis presented in Figure 4.

**Fact#2 - HVSR data**

A further confirmation of the overall consistency of the above-presented analyses is given by the Horizontal-to-Vertical Spectral Ratio acquired on the site. Figure 7 reports the observed and modeled HVSR curves (the modeled curve refer to the average model obtained from the final Pareto front models reported in Figure 4).

![Figure 7](image)

**Figure 7.** Observed and modeled (average model from the Pareto front models obtained from the second interpretative hypothesis) Horizontal-to-Vertical Spectral Ratio (from microtremors).
Final remark

It can be noted that (also in accordance with the theory – e.g. Dal Moro, 2012; Dal Moro and Marques Moura, 2012) Love waves show a much simpler “phenomenology” which results extremely useful (and under some circumstances even necessary) to properly interpret Rayleigh-wave dispersion.

It must be finally underlined that the joint analysis of Rayleigh and Love waves plus HVSR via forward modeling (i.e. the user personally modifies the $V_s$ and thickness values while seeking for a good overlap between observed and modeled data) can be done in the "Velocity Spectra, Modeling & Picking" panel of winMASW (see video tutorials available from the www.winmasw.com site - area "publications").

References

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Joint Analysis of Surface Waves (Dal Moro G.), Graz (Austria), 9-13 July 2012, (Mini-Symposium Surface and Interface Acoustic Waves in Solids, 8th European Solid Mechanics Conference), invited speaker


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